ATTENTION MODELS, WORLD MODELS, AND GOING BEYOND WORLD UNDERSTANDING

François Fleuret

One key technology of "modern AI" are the convolutional models.

- They are powerful function approximators.
- $\circ~$ Scale well with data set size and computation. **lower-order --,. higher-order .-,. ~ .grandmother**
- Fitting for hierarchical signal structures.

Neocognitron [\(Fukushima, 1980\)](#page-88-0) $\overline{}$

A convolution applies the **same linear operation** at every location in the signal.

Such mechanisms are very efficient for image or sound processing where

- the signal is stationary, and
- local structures are very informative.

However some tasks involve more than composing local structures, e.g. translation:

> "**An apple** that had been on the tree in the garden for weeks had finally been **picked up.**"

> "**Une pomme** qui était sur l'arbre du jardin depuis des semaines avait finalement été **ramassée**."

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It has motivated the development of **attention-based processing** to transport information from parts of the signal to other parts dynamically identified.

A standard attention layer takes as input two sequences X and X' and computes

$$
Q = XW^{Q^{\top}}
$$

\n
$$
K = X'W^{K^{\top}}
$$

\n
$$
V = X'W^{V^{\top}}
$$

\n
$$
A = softmax_{row} \left(\frac{QK^{\top}}{\sqrt{d}}\right)
$$

\n
$$
Y = AV
$$

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When $X = X'$, this is **self attention**

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When $X = X'$, this is **self attention**, otherwise **cross attention.**

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```
Sequential(
  (0): Conv1d(1, 64, kernel size=(5,), stride=(1,), padding=(2,))
  (1): ReLU()(2): Conv1d(64, 64, kernel_size=(5,), stride=(1,), padding=(2,))
  (3): ReLU()(4): Conv1d(64, 64, kernel_size=(5,), stride=(1,), padding=(2,))
  (5): ReLU()
  (6): Conv1d(64, 64, kernel_size=(5,), stride=(1,), padding=(2,))
  (7): ReLU()
  (8): Conv1d(64, 1, kernel size=(5,), stride=(1,), padding=(2,)))
```


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The poor performance of this model is not surprising given its inability to channel information from "far away" in the signal.

More layers, global averaging, or fully connected layers could possibly solve the problem. However it is more natural to equip the model with the ability to fetch information from parts of the signal that it actively identifies as relevant.

This is exactly what an **attention layer** does.

```
class SelfAttentionLayer(nn.Module):
   def __init__(self, in_dim, out_dim, key_dim):
        super() . _{init} ( )self.convQ = nn.Conv1d(indim, kevdim, kernel size = 1, bias = False)self.convK = nn.Conv1d(in dim, kev dim, kernel size = 1, bias = False)self.convV = nn.Conv1d(in dim, out dim, kernal size = 1, bias = False)def forward(self, x):
       Q = self.comv(Q(x))K = self.comV K(x)V = self.comv/V(x)A = torch.einsum('nct.ncs->nts', Q, K).softmax(2)
       y = torch.einsum('nts,ncs->nct', A, V)
       return y
```

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Sequential(
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  (1): ReLU()
  (2): Conv1d(64, 64, kernel_size=(5,), stride=(1,), padding=(2,))
  (3): ReLU()(4): SelfAttentionLayer(in_channels=64, out_channels=64, key_channels=64)
  (5): Conv1d(64, 64, kernel size=(5,), stride=(1,), padding=(2,))(6): ReLU()(7): Conv1d(64, 1, kernel_size=(5, ), stride=(1, ), padding=(2, ))
)
```


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Transformers

The original transformer combines a stack of self-attention layers in an encoder, and a stack of causal self-attention and cross-attention layers in a decoder.

[\(Vaswani et al., 2017\)](#page-88-1)

The original transformer combines a stack of self-attention layers in an encoder, and a stack of causal self-attention and cross-attention layers in a decoder. The GPT model keeps only a causal encoder.

Transformers

Large language models have been shown to exhibit some "zero shot learning" capabilities when they are properly "primed" [\(Brown et al., 2020\)](#page-88-2).

For instance using Hugging Face's gpt2 model with 120M parameters, we can get these sentence completions, where the generated parts are in bold:

I: water boils at 100 degrees, O: physics, I: the square root of two is irrational, O: mathematics, I: the set of prime numbers is infinite, O: mathematics, I: gravity is proportional to the mass, O: **physics,**

I: water boils at 100 degrees, O: physics, I: the square root of two is irrational, O: mathematics, I: the set of prime numbers is infinite, O: mathematics, I: squares are rectangles, O: **mathematics,**

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I: I love apples, O: positive, I: music is my passion, O: positive, I: my job is boring, O: negative, I: frozen pizzas are awesome, O: **positive,**

I: I love apples, O: positive, I: music is my passion, O: positive, I: my job is boring, O: negative, I: frozen pizzas taste like cardboard, O: **negative,**

Transformers

[\(Dosovitskiy et al., 2020\)](#page-88-3)

GPTS FOR WORLD MODELS

World model

Reinforcement learning is in general sample-inefficient, since learning a policy

$$
\pi: S \to A
$$

$$
X_t \mapsto A_t
$$

to maximize an accumulated reward requires to propagate back rewards from the sparse states where it occurs.
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to maximize an accumulated reward requires to propagate back rewards from the sparse states where it occurs.

A solution is to train a **world model** that is a model of

 $P(X_{t+1} | A_s, X_s, s \leq t)$

that does not need a sophisticated "lookahead", and then to train an agent in that model. For image-based RL, this is close to next-frame prediction.

World model

We focus on training with limited samples, in particular Atari with 100k frames.

[\(fleuret.org/vid\)](./vid/iris/iris/IRIS_large.gif)

This is a standard benchmark with varied dynamics and "semantics".

We developed IRIS, that combines a discrete autoencoder (VQ-GAN) with a GPT.

The agent is an LSTM-based model entirely trained in generated episodes.

Repeat 500 times:

- From the current state, execute 200 steps in the environment (≃ 3s) with an ϵ -greedy policy.
- Add the frames/actions to the sample set.
- Improve the autoencoder and the GPT world model from the collected samples.
- Optimize the value network and the policy from generated trajectories.

[\(fleuret.org/vid\)](./vid/iris/iris/IRIS_accuracy.gif)

[\(fleuret.org/vid\)](./vid/iris/iris/IRIS_long_time.gif)

The core issue with IRIS is the number of tokens per frame. We had to use 16 tokens, leading to a substantial degradation of performance.

We address this with ∆-IRIS, which encodes a frame as a small number of discrete tokens **given the four previous frames and actions.**

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We address this with ∆-IRIS, which encodes a frame as a small number of discrete tokens **given the four previous frames and actions.**

A GPT generates the z_t^k , but attend to image tokens produced with the z_s^k , $s < t$.

GPT-generated ∆-tokens

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GPT

GPT vs. random tokens

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Two key realizations:

◦ **discrete tokens are needed for prediction** to benefit from cross-entropy and deal with multi-modality, but past information can be processed and integrated into **continuous tokens.**

Two key realizations:

- **discrete tokens are needed for prediction** to benefit from cross-entropy and deal with multi-modality, but past information can be processed and integrated into **continuous tokens.**
- A perfect differential encoder would generate **totally random delta embeddings** and be in itself a world model.

Crafter 10M frames, ∆-IRIS solves 17 out of Crafter's 22 tasks.

Atari 100k frames, ∆-IRIS is SOTA for # Superhuman.

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- The computational abilities of auto-regressive models are amazing.
- The inductive bias of transformers as auto-regressive models is amazing.
- Are the computational abilities leveraged by LLMs?
- Should spatial and motor understanding be learned first as a foundation for abstract reasoning?

BEYOND WORLD UNDERSTANDING

[Warning! Computer scientist's armchair philosophy] How can we go beyond reasoning tools for planing in the environment? How to get high-level abstract "thinking" (mathematics, humor, poetry?) [Warning! Computer scientist's armchair philosophy]

How can we go beyond reasoning tools for planing in the environment?

How to get high-level abstract "thinking" (mathematics, humor, poetry?)

An hypothesis is that this happens through reasoning challenges in social competition.

This allows to transfer capabilities from one model to the others and could trigger a "cognitive arm race" and go beyond reasoning tools to model the environment.

If, to operate in its environment, an agent has to solve

$$
\Set{\blacktriangle, \blacksquare, \blacktriangle, \blacksquare} \mapsto (\blacktriangle, \blacktriangle), (\blacksquare, \blacksquare)
$$

it can do it by being only "color aware"

$$
\{\textcolor{gray}{\bullet},\textcolor{gray}{\bullet},\textcolor{gray}{\bullet},\textcolor{gray}{\bullet}\} \mapsto (\textcolor{gray}{\bullet},\textcolor{gray}{\bullet}),(\textcolor{gray}{\bullet},\textcolor{gray}{\bullet})
$$

or "shape aware"

$$
\{\ \blacktriangle\ ,\ \blacksquare\ ,\ \blacktriangle\ ,\ \blacksquare\ \} \mapsto (\triangle\ ,\ \triangle\),\ (\blacksquare\ ,\ \blacksquare\).
$$

A "shape aware" agent could craft a challenge that those who are not would fail

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$\{ \blacktriangle, \blacksquare, \blacktriangle, \square \} \mapsto (\blacktriangle, \blacktriangle), (\blacksquare, \blacksquare)$

and similarly for a "color aware" agent

To test this experimentally we use GPT models as agents and "quizzes" as elements of reasoning.

Let $\mathcal{G}=\{$ "white", "red", $\dots\}^{10\times10}$ be the set of colored 10 \times 10 grids. A quiz consists of four grids that should be interpreted as

 $(A, f(A), B, f(B))$

where $f: G \to G$ is a latent function.

Such a quiz should be interpreted as a prompt composed of three 10×10 colored cell grids, and a solution which is a single grid.

We define seven "tasks", each of them being a distribution of "world quizzes", such that the solution is unique given the prompt.

World and Culture quizzes

"Half Fill"

"Translate"

"Grow"

"Motion"

We differentiate:

- "world quizzes" whose distributions are those predefined tasks, and play the role of the cognitive challenges from the environment, and
- "culture quizzes" which are generated by agents, and whose role is to prove that its author masters certain concepts that its opponents do not.

Experiments are done with a 37 millions parameter GPT trained from scratch.

- dim_model: 512
- dim_keys: 64
- dim_hidden: 2048
- nb_heads: 8
- nb_blocks: 12

Given a quiz composed of three 10 \times 10 grids for the prompt and one 10 \times 10 grid for the solution

>

we can represent the prompt as a sequence of 300 tokens

 $R_0(A), \ldots, R_9(A), R_0(f(A)), \ldots, R_9(f(A)), R_0(B), \ldots, R_9(B)$

and the solution as a sequence of 100 tokens

 $R_0(f(B)), \ldots, R_9(f(B))$

We can train a GPT on a complete sequence composed of the prompt followed by the solution, and then use it to "solve" a quiz, that it to generate the solution given the prompt.

We re-sample the training examples at every epoch, so there is no over-fitting. The accuracy of a GPT trained on all the tasks combined gets easily above 99%. Our main objective is to **generate new meaningful quizzes**, which are both outside the domain of the training examples and consistent with them.

A GPT can produce a full quiz by generating all the tokens without conditioning.

However, if the GPT is properly trained, such a quiz follows the data-distribution, and there would be no novelty beside unstructured sampling noise.

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However, if the GPT is properly trained, such a quiz follows the data-distribution, and there would be no novelty beside unstructured sampling noise.

We propose to train N GPTs in parallel, to generate quizzes with structured noise, and to "validate" and keep a generated quiz **only if exactly** N − 1 **GPTs solve it correctly.**
To generate quizzes with a structured noise, we

- 1. generate a solution at high temperature,
- 2. generate a consistent prompt at low temperature, and
- 3. re-generate the solution given this prompt at low temperature.

That way we get a quiz that may be slightly outside the support of the training samples, but it should be "functionally consistent."

Hence we also need to generate prompts given solutions. To do that we train the model on two types of sequences, with additional tokens to indicate the direction.

1. Generate a solution at high temperature

2. Generate a prompt at low temperature

3. Re-generate the solution at low temperature

We then evaluate the quiz with each of the N models and count the number of correct prediction M and "validate" the new quiz only if $M = N - 1$.

The global loop proceeds as follows at each iteration:

- 1. If all the models have an accuracy above a threshold (e.g. 0.95), generate culture quizzes and add them to the culture.
- 2. Select the model with the lowest accuracy, build a training set with up to 50% culture quizzes, and completed with fresh world quizzes, proceed with one epoch of training with this set.

Various number of rectangles.

"Half Fill" with complex coloring.

"Half Fill" with non-rectangular shapes.

"Half Fill" with two colors or two rectangles to fill.

"Translate" with the moving part occluded.

"Translate" with the moving part occluded.

"Frame" and "Half fill" combined.

"Translate" with the moving part occluded.

"Frame" and "Half fill" combined.

"Frame" generalized to non-rectangular shapes.

"Detect" with location markers colored according to the rectangle colors.

Questions?

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