Deep learning

1.4. Tensor basics and linear regression

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- A 1d tensor is a vector (e.g. a sound sample),
- A 2d tensor is a matrix (e.g. a grayscale image),
- A 3d tensor can be seen as a vector of identically sized matrix (e.g. a multi-channel image),
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- etc.

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Manipulating data through this constrained structure allows to use CPUs and GPUs at [near] peak performance.

The "dimension" of a vector in linear algebra is its number of coefficients, while the "dimension" of a tensor is the number of indices to specify one of its coefficients.

E.g. an element of $\mathbb{R}^3$ is a three-dimension vector, but a one-dimension tensor.

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A key specificity of PyTorch is the central role of autograd to compute derivatives of *anything* ! We will come back to this.

```
>>> x = torch.empty(2, 5)
>>> x.size()
torch.Size([2, 5])
>>> x.fill_(1.125)
tensor([[ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250],
        [ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250]])
>>> x.mean()
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>>> x.std()
tensor(0.)
>>> x.sum()
tensor(11.2500)
>>> x.sum().item()
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Reading a coefficient returns a 0d tensor.

```
>>> x = torch.tensor([[11., 12., 13.], [21., 22., 23.]])
>>> x[1, 2]
tensor(23.)
```

PyTorch provides operators for component-wise and vector/matrix operations.

```
>>> x = torch.tensor([ 10., 20., 30.])
>>> y = torch.tensor([ 11., 21., 31.])
>>> x + y
tensor([ 21.,  41.,  61.])
>>> x * y
tensor([ 110.,  420.,  930.])
>>> x**2
tensor([ 100.,  400.,  900.])
>>> m = torch.tensor([[ 0., 0., 3. ],
...                   [ 0., 2., 0. ],
...                   [ 1., 0., 0. ]])
>>> m.mv(x)
tensor([ 90.,  40.,  10.])
>>> m @ x
tensor([ 90.,  40.,  10.])
```

And as in NumPy, the `:` symbol defines a range of values for an index and allows to slice tensors.

```
>>> import torch
>>> x = torch.randint(10, (2, 4))
>>> x
tensor([[8, 7, 6, 6],
        [5, 0, 4, 8]])
>>> x[0]
tensor([8, 7, 6, 6])
>>> x[0, :]
tensor([8, 7, 6, 6])
>>> x[:, 0]
tensor([8, 5])
>>> x[:, 1:3] = -1
>>> x
tensor([[ 8, -1, -1,  6],
        [ 5, -1, -1,  8]])
```

PyTorch provides interfacing to standard linear operations, such as linear system solving or eigen-decomposition.

```
>>> y = torch.randn(3)
>>> y
tensor([ 1.3663, -0.5444, -1.7488])
>>> m = torch.randn(3, 3)
>>> q = torch.linalg.lstsq(m, y).solution
>>> m@q
tensor([ 1.3663, -0.5444, -1.7488])
```

Example: linear regression

Given a list of points

$$(x_n, y_n) \in \mathbb{R} \times \mathbb{R}, \;\; n = 1, \dots, N,$$

can we find the affine function

$$f(x; a, b) = ax + b$$

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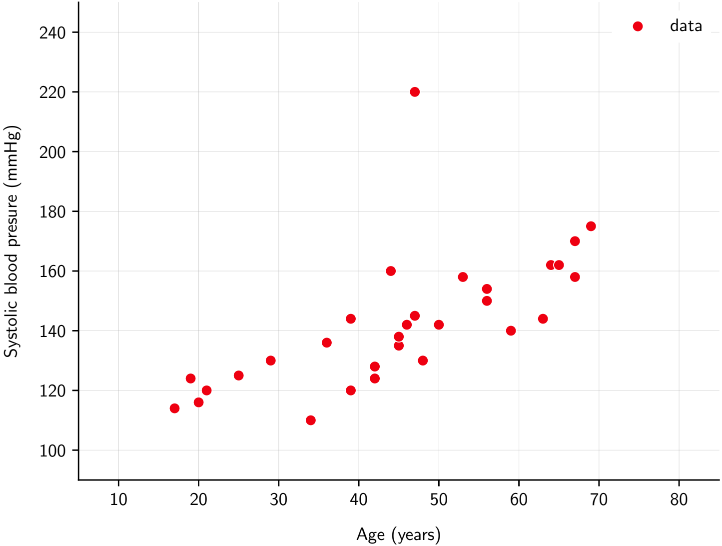
Such a model would allow to predict the y associated to a new x, simply by calculating $f(x; a, b)$.

```
bash> cat systolic-blood-pressure-vs-age.dat
39  144
47  220
45  138
47  145
65  162
46  142
67  170
42  124
67  158
56  154
64  162
56  150
59  140
34  110
42  128
/.../
```


data
Systolic blood presure (mmHg)
240
220
200
180
160
140
120
100
10
20
30
40
50
60
70
80
Age (years)

$$\underbrace{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_N & y_N \end{pmatrix}}_{\text{data} \in \mathbb{R}^{N \times 2}}$$

$$\underbrace{\begin{pmatrix} x_1 & 1.0 \\ x_2 & 1.0 \\ \vdots & \vdots \\ x_N & 1.0 \end{pmatrix}}_{x \in \mathbb{R}^{N \times 2}} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{\alpha \in \mathbb{R}^{2 \times 1}} \simeq \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{y \in \mathbb{R}^{N \times 1}}$$

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```
import torch, numpy

data = torch.tensor(numpy.loadtxt('systolic-blood-pressure-vs-age.dat'))
nb_samples = data.size(0)

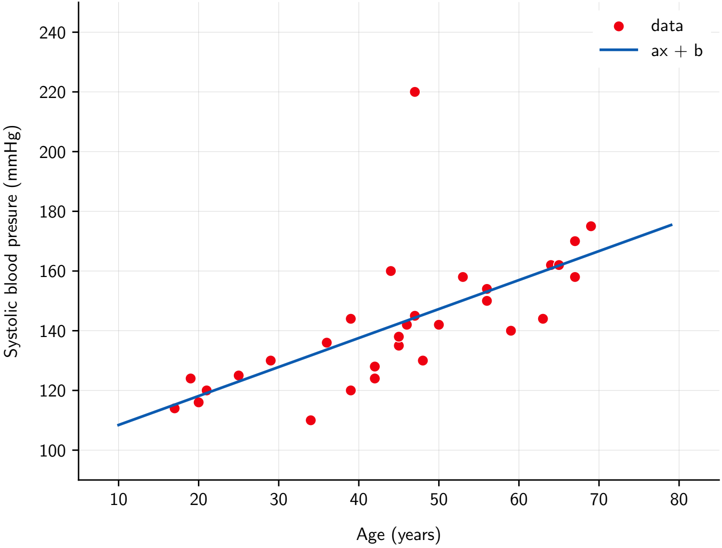
x, y = torch.empty(nb_samples, 2), torch.empty(nb_samples, 1)

x[:, 0] = data[:, 0]
x[:, 1] = 1

y[:, 0] = data[:, 1]

alpha = torch.linalg.lstsq(x, y).solution

a, b = alpha[0, 0].item(), alpha[1, 0].item()
```


data
ax + b
Systolic blood presure (mmHg)
240
220
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70
80
Age (years)

The end